13th GAMM-Seminar Kiel

on

Numerical Treatment of Multi-Scale Problems

Kiel, January 24th to 26th, 1997

Eds.: W. Hackbusch (Kiel), G. Wittum (Stuttgart)

The GAMM Committee 'Efficient numerical methods for pdes' in cooperation with the Christian-Albrechts-Universität Kiel organises the

13th GAMM-Seminar Kiel

on

Numerical Treatment of Multi-Scale Problems

Kiel, January 24th to 26th, 1997

Chairmanship:	Wolfgang Hackbusch (Kiel), Gabriel Wittum (Stuttgart)
Location:	Mathematisches Seminar und Institut für Informatik und Praktische Mathematik, Universität Kiel, Germany
Conference fee:	DM 70.- (to be paid after arrival)
Local organization:	Jens Burmeister, Tel.: ++49-431-880-4462, Fax: ++49-431-880-4054, Email: `jb@numerik.uni-kiel.de`
WWW-site:	`http://www.numerik.uni-kiel.de/gamm.html`

Programme for Thursday, January 23rd, 1997:
Those arriving in Kiel on Thursday are invited to meet about 19.00 h in a restaurant and pub called 'DÜPPEL 88' (Düppelstr. 88, 24105 Kiel).

Friday, January 24th, 1997

9.00	Opening
9.15 - 9.45	**S.A. Sauter (Kiel):** Composite Finite Elements for coarse-level discretizations of PDEs with essential boundary conditions
9.50 - 10.20	**V. Bulgakov, G. Kuhn (Erlangen):** Application of the multi-level aggregation method to structural analysis problems

Coffee-break

10.55 - 11.20	**F.A. Bornemann, R. Krause (Stuttgart):** Classical and Cascadic Multigrid - A Methodical Comparison
11.25 - 11.55	**M. Griebel, S. Knapek (Bonn):** A Multigrid Homogenization Method
12.00 - 12.30	**G. W. Zumbusch (Bonn):** Multi-grid for plate bending in scattered data interpolation

Lunch

14.30 - 14.50	**T. Rother, H. Kasper, O. Kolditz (Hannover):** Geometric modelling of complicated hydrogeologic structures
14.55 - 15.25	**O. Filippova, D. Haenel (Duisburg):** Computation of gas-particle flow through filters by a lattice-Boltzmann method
15.30 - 15.50	**M. Lambrecht, O. Knoth (Leipzig):** Discretization of Advection on an ARAKAWA-C-grid with Internal Boundaries

Coffee-break

16.30 - 17.00	**J. Piquet, X. Vasseur (Nantes, France):** Three-dimensional multigrid based pressure solver for the computation of the flow around the HSVA Tanker
17.05 - 17.35	**J. Tausch, J. White (Cambridge, USA):** BEM Solution of Laplace's Equation in Multi-LayeredMedia with High Permittivity Ratios

Saturday, January 25th, 1997

9.00 - 9.30	**W. Dahmen, R. Schneider (Chemnitz):** Biorthogonal Wavelet Bases on Domains and Manifolds
	Coffee-break
10.00 - 10.30	**A. Kunoth (Berlin):** On the Treatment of Boundary Conditions in Multiscale Methods for Elliptic P.D.E.'s
10.35 - 11.05	**B. Gottschlich-Müller (Aachen):** Multiresolution Schemes for Conservation Laws
11.10 - 11.40	**S. Müller (Aachen):** A robust multigrid solver for convection-dominated problems
	Lunch
14.15 - 14.45	**B. Amaziane, B. Ondami (Pau, France):** Numerical Approximations of a Second Order Elliptic Equation with Highly Oscillating Coefficients
14.50 - 15.20	**Johan Walden (Uppsala, Sweden):** Filter Bank Methods for Hyperbolic PDEs
15.25 - 15.55	**B. N. Khoromskij, G. Wittum (Stuttgart):** Robust iterative methods by interface reduction for solving highly anisotropic elliptic equations
	Coffee-break
16.30 - 17.00	**N. Neuss (Heidelberg):** Optimal error estimates and multiscale applications
17.05 - 17.35	**Ch. Lubich (Tübingen):** Discretizations of multi-scale problems in ODEs
17.40 - 18.00	**C.-D. Munz, S. Roller, E. Sonnendrücker (Karlsruhe):** A Numerical Approach to Multiple Scale Problems Based on Asymptotic Analysis
	18.05 h Short presentations
18.05 - ...	**D. Funaro, (Modena, Italy):** Spectral Elements for Advection-Diffusion Equations
	H. Holke, O. Knoth (Leipzig): Atmospheric Flows on Locally Refined Grids
	O. Knoth (Leipzig): A multilevel pressure solver for flows around buildings
	18.30 h Reception

Sunday, January 26th, 1997

9.00 - 9.30 **M. Dryja (Warsaw, Poland):** The additive Schwarz method for finite element anisotropic elliptic problems

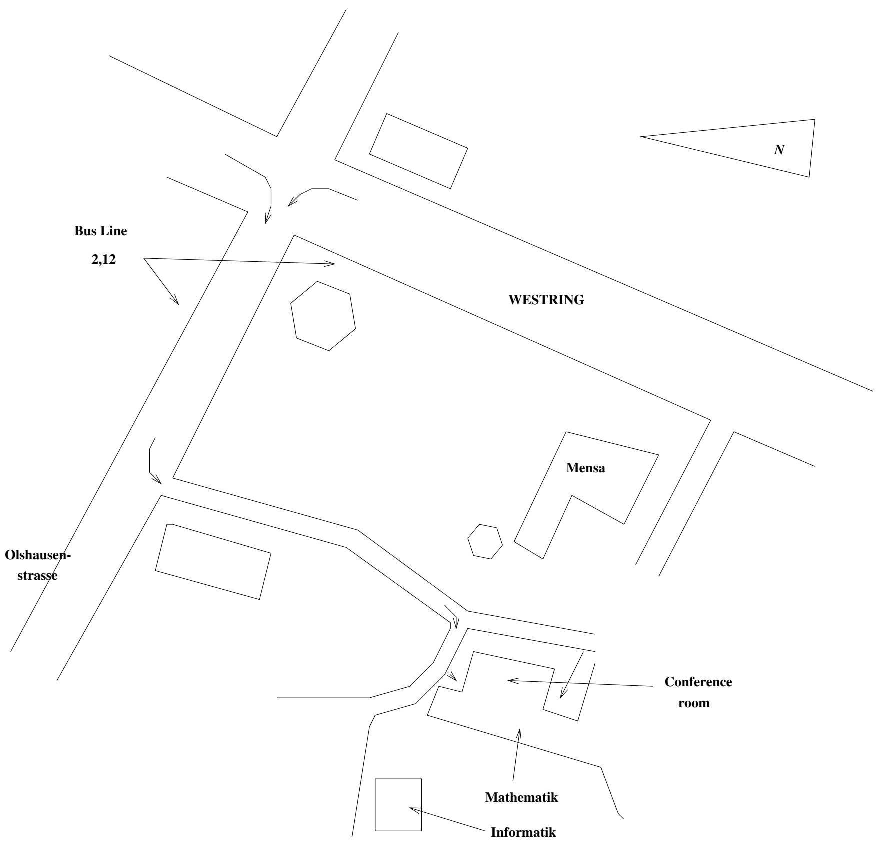
9.35 - 9.55 **A. Auge, G. Lube, F.C. Otto (Göttingen):** A note on an adaptive nonoverlapping domain decomposition method for elliptic problems

Coffee-break

10.20 - 10.50 **F. Ihlenburg (Austin, USA):** On the correct and reliable resolution of different scales in finite element analysis of exterior Helmholtz problems

10.55 - 11.25 **J. Schöberl (Linz, Austria):** Anisotropic Elements in Linear Elasticity

End of 13th GAMM-Seminar Kiel



How to find the conference room (Bus lines 2 and 12):

Collection of Abstracts:

Collection of Abstracts:

Numerical Approximations of a Second Order Elliptic Equation with Highly Oscillating Coefficients

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Numerical approximations are considered for the solutions of a class of elliptic equations in divergence form in the case of periodic homogenization. Specifically, we consider boundary value problems of the form

$$(P_\varepsilon) \quad \begin{cases} -\nabla.\left(K^\varepsilon(x)\nabla u_\varepsilon\right) = f & in \quad \Omega \\ u_\varepsilon = 0 \quad on \quad \Gamma = \partial\Omega \end{cases}$$

where Ω is a bounded domain in IR^2 with polygonal boundary, f is a function in $L^2(\Omega)$, and where $K^\varepsilon(x) = K\left(\frac{x}{\varepsilon}\right)$ with $K\epsilon\left(L^\infty(Y)\right)^4$ is Y-periodic and satisfies

$$0 < \alpha\left|\overrightarrow{\zeta}\right|^2 \leq K(y)\overrightarrow{\zeta}\cdot\overrightarrow{\zeta} \leq \beta\left|\overrightarrow{\zeta}\right|^2 \quad \forall\overrightarrow{\zeta}\epsilon IR^2 \quad a.e.\ in\ y\epsilon Y$$

where $0 < \alpha \leq \beta < \infty$ are constants. ε denotes the parameter involved in the periodic homogenization.
It is proposed to use both conforming and mixed finite elements to solve the problem (P_ε). Error estimates depending on ε and h (mesh size) are established for each method. It's shown that certain mixed finite element methods are more accurate and robust than conforming finite element method for the class of problems studied.
Numerical simulations for one-dimensional problem comparing the approximations obtained by these methods to the approximation of the homogenized problem are presented.

Application of the multi-level aggregation method to structural analysis problems

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Recently developed multi-level aggregation method is a method of multigrid type. It is applicable to finite element structural analysis problems ranging from the boundary value theory of elasticity to reticulated frame and truss problems. A special coarsening strategy based on the rigid body motion concept is employed which allows the use of arbitrary displacement formulated finite element model. The hierarchy of aggregation levels is constructed automatically on the basis of information obtained from the finite element stiffness matrix coefficients. The general algebraic scheme of the method consists of the external iterative procedure with multi-level preconditioning. The method has been implemented as an industrial equation solver FEAGS which can be used in large comprehensive finite element software packages. Numerical experiments and industrial computations show that its convergence and solution time in many cases are far superior to modified ICCG and direct methods. Such numerical examples as off-shore platforms, fly wheels, high-rise buildings, gravity dam structures, bus frame and others demonstrate efficiency of the solver.

[**1**] V.E. BULGAKOV, G. KUHN: *High-Performance Multi-level Iterative Aggregation Solver for Large Finite-Element Structural Analysis Problems*, Int. J. Numer. Meth. Eng., 38, 3529-3544 (1995).

The additive Schwarz method for finite element anisotropic elliptic problems

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A finite element approximation of anisotropic second order elliptic problems on a rectangular region is discussed. The coefficients are piecewise constant on rectangular subregions of the original region. The additive Schwarz method (AMS) with minimal overlap for solving the resulting systems is described and analyzed. It uses three coarse spaces, standard and two special.

A rate of convergence of the method is of the order of $(H/h)^{\frac{1}{2}}$ when cg is used. Here H and h are parameters of the coarse and fine triangulation. The rate of convergence of the method is independent of the coefficients, i.e. their discontinuity and anisotropy.

Spectral Elements for Advection-Diffusion Equations

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We would like to describe a way to approximate boundary-value problems by spectral elements, which is particularly suited for advection-diffusion equations, where the second-order diffusive terms are largely dominated by the first-order transport terms.
First of all, we need a good solver for the case of the single domain (i.e. the usual square in 2-D or the cube in 3-D). We consider the technique suggested in [1], where the equation is collocated at a set of points obtained by shifting upwind (with respect to the direction

of the flux individuated by the advective terms) the standard Legendre nodes. This procedure is interesting for two reasons: it provides *stabilized* approximated solutions (not affected by spurious oscillations) and allows the construction of very effective finite-differences preconditioners, giving a fast solution of the systems by an iterative solver, with a number of iterations not depending on the polynomial degree and the size of the advective terms.

The second issue concerns the implementation of the spectral elements. An iterative domain-decomposition algorithm reduces the global solution to a sequence of subproblems in each single domain. The technique we adopted is standard. In practice, we impose Dirichlet boundary conditions at the interfaces, and update the interface values according to the error of the normal derivatives of the multidomain solution across the interface sides. A speed up of the convergence is obtained by a suitable tridiagonal preconditioner applied to the values at the nodes of each interface side. This is the crucial part of the implementation. We present a preconditioning matrix capable to handle advection dominated equations, resulting in a convergence behavior which seems to be independent, as pointed out by numerical experiments, of both the polynomial degree and the size of the advective terms. To this purpose a special set of grid points has to be taken on each interface side. These nodes are located upwind with respect to the direction of the tangential component of the flux along the interface.

The method has been successfully experimented and in [2] the whole procedure is explained and supplied with a lot of test cases, including the solution of incompressible Navier-Stokes equations.

[1] *Funaro D.*, A New Scheme for the Approximation of Advection-Diffusion Equations by Collocation, SIAM J. Numer. Anal., Vol.30, n.6(1993), pp.1664-1676.

[2] *Funaro D.*, *Spectral Elements for Transport Dominated Equations*, to appear.

Multiresolution Schemes for Conservation Laws

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In recent years a large variety of high order schemes for the simulation of flows were developed. Neglecting dissipative effects these problems are described by hyperbolic conservation laws. Their physical meaningful solution can be approximated by finite–volume methods in conservative form. These schemes involve complex flux evaluations in order to resolve shocks and contact discontinuities. However, such discontinuities only occur on lower dimensional manifolds of the flow field. Only there it is necessary to employ the expensive scheme while elsewhere a simple finite–difference scheme is sufficient. In order to reduce the number of flux evaluations in these domains as much as possible, A. Harten developed a general multiresolution strategy for the one–dimensional case. This results in a new method which is both adaptive and hybrid. It replaces the expensive flux evaluation by interpolation of surrounding fluxes in domains where the solution is smooth. Harten's concept can be realized by using biorthogonal wavelets due to A. Cohen, I. Daubechies and J.–C. Feauveau. The generalization to multi–dimensional problems was also possible. Finally, some numerical results for the two–dimensional Euler equations are presented.

A Multigrid Homogenization Method

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For many applications the coefficients of the mathematical model incorporate strong variations on several length scales. A typical example is the diffusion coefficient in porous media. In general the multiscale asymptotic analysis for the determination of so called effective values for the modeling of approximate coarse scale problems is far to costly. Therefore simple averaging techniques like arithmetic, harmonic mean and renormalization procedures are used, which are computationally efficient, but inaccurate for coefficients with strong variations.

We present a method to approximately determine the effective diffusion coefficient on a coarse scale level. We apply the Galerkin approximation with operator- or matrix-dependent prolongations and Schur complement approximations. This leads to energy-dependent averaging procedures and to averaged equations which describe the large-scale behavior of the problem in discrete form.

Limitations of the usage of well known matrix-dependent prolongations in this approach are discussed. Techniques such as smoothing of the prolongation are given, which improve the presented method significantly.

We explain our modelling approach derived from multigrid, discuss its properties and compare it with other averaging techniques.

Computation of gas-particle flow through filters by a lattice-Boltzmann method

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A numerical concept can be presented the investigation of the interaction of fluid flow, particle transport and particle deposition in fibrous filters. Deposited particles (spheres)

change the shape of filter material in a very irregular manner, which influences again the flow and the following deposition. The particle transport is calculated by Lagrangian integration together with a model for deposition. Essential difficulties arise with the computation of fluid flow around complex boundaries at low Reynolds numbers and Mach numbers near zero. The Lattice-Boltzmann Method has found to be a suitable to deal with such flows. Presently the gaskinetical Krook (BGK) approximation of the Boltzmann equation is solved for discrete space and molecular velocity on an equidistant grid. By multi-scale analysis it is shown that this model approximates the Navier-Stokes equations of an incompressible fluid. Based on the analysis boundary formulations are derived for fitting arbitrary solid boundaries with second order accuracy. The solution concept is demonstrated by several 2-D and 3-D results.

Atmospheric Flows on Locally Refined Grids

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Meshes for discretizing anelastic atmospheric transport equations should allow variable cell sizes (in horizontal and vertical direction) to include processes acting on different scales. Currently used (uniform) meshes permit vertical layers of different thicknesses and differing mesh width's for each horizontal layer.

Sometimes a better spatial resolution is needed (because of large gradients of emissions or surface, for example). In these cases smoothed locally refined grids in terrain–following coordinates are adequate. One cell may have more than one neighbour cell in each direction, then. Suitable data structures and schemes for discretization have been developed for such meshes. Those schemes have to be exact at least for constant pressure and velocity fields and have to conserve mass. This can be achieved by a suitable coarsening of the data given at the finest level of discretization (e.g. surface data and terms describing the coordinate transformation) and by suitable discretization at faces belonging to cells of different sizes.

A multiscale finite element method for computing highly oscillatory solutions in PDE's

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We study a multiscale finite element method for solving a class of physical problems arising from composite materials, flows in porous media, and turbulent transport which contain many spatial scales. The method is designed to efficiently capture the large scale behavior of the solution without resolving all the small scale features. This is accomplished by constructing the multiscale finite element base functions that are adaptive to the local property of the differential operator. Our method is applicable to general multiple-scale problems without restrictive assumptions on scale separation and periodicity. The construction of the base functions is fully decoupled from element to element; thus, the method is perfectly parallel and is naturally adapted to massively parallel computers. For the same reason, the method has the ability to handle extremely large degrees of freedom due to highly heterogeneous media, which are otherwise intractable by conventional finite element (difference) methods. In contrast to some empirical numerical upscaling methods, the multiscale method is systematic, self-consistent, and it can be analyzed rigorously. In the case of periodic oscillatory structures, convergence of our method has been established and the rate of convergence is shown to be independent of the small scales of the solution. We demonstrate the accuracy and efficiency of our method through extensive numerical experiments, which include problems with random coefficients and problems of continuous scales. Steady conduction through fiber composites and flows through random media with normal and fractal porosity distributions will also be considered. Parallel implementation and performance of the method are also addressed.

On the correct and reliable resolution of different scales in finite element analysis of exterior Helmholtz problems

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Exterior problems for time-harmonic wave propagation are inherently multiscalar. Firstly, the numerical method has to resolve the finite scale of the source/ obstacle as well as the infinite scale of the exterior region. In particular, it is important that the far field behavior of the exact model it represented correctly in the approximate model. Secondly, the length of the radiated or incident/scattered waves may be small compared to the size of the source/obstacle. The relation small wavelength/ large scattering object leads to Helmholtz equations with large (non-dimensional) wavenumber. It can be shown that the stability of the numerical model deteriorates with the size of the wavenumber. This has an important effect on the accuracy and reliability of the numerical model. In our talk, we address both aspects of the problem. Refering to the incorporation of the infinite scale, we present new results from a convergence analysis of a coupled FEM-infinite element method (with L. Demkowicz). Regarding the resolution of small wavelengthes, we give results from our analysis of the h-p-method of the FEM (with I. Babuška).

Robust iterative methods by interface reduction for solving highly anisotropic elliptic equations

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We propose and analyze the robust and asymptotically optimal iterative method for solving an interface reduction to anisotropic elliptic equations with piecewise constant coefficients, where the direction of anisotropic behaviour may change within a domain. Jumps of anisotropy coefficients align with the coarse mesh decomposition. The construction is essentially based on using of an exotic nonconformal coarse mesh space which resolves qualitatively the interior boundary layers near the interface. Our method extends the familiar Bramble-Pasciak-Schatz preconditioner to the case of highly anisotropic equations. Under certain assumptions on the diffusion coefficients behaviour, a stable multilevel method is cosidered to solve the underlying coarse mesh problem. The results of numerical experiments are presented.

A multilevel pressure solver for flows around buildings

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A discretization of the *grad* and *div* operators on a cartesian grid are given as a part of an incompressible Navier–Stokes–Solver for the computation of flow patterns around buildings. The buildings are described on a three–dimensional cartesian grid by dividing the cartesian cells in three groups, free cells, body cells, and cut cells. Cut cells which are partially intersected by a building are characterized by the free volume part of the cell and the free area part of each of its six faces. For a cell centered piecewise constant pressure and a staggered arrangement of the velocity components the discretization of both operators is constructed by the use of the cut cell information and analog information on the shifted grids. The pressure correction is then computed on a sequence of nested cartesian grids where the operators are constructed for each grid level.

We compare a cascadic CG-like method and V–cycle multigrid algorithm for different building configurations.

Classical and Cascadic Multigrid - A Methodical Comparison

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Using the full multigrid method without any coarse grid correction steps but with an a posteriori control of the number of smoothing iterations was shown by Bornemann and Deuflhard to be an optimal iteration method with respect to the energy norm. They named this kind of multigrid iteration the cascadic multigrid method. However, numerical examples with linear finite elements raised serious doubts, wether the cascadic multigrid method can be made optimal with respect to the L^2 - norm. It will be shown that the cascadic multigrid method cannot be optimal for linear finite elements. The cascadic multigrid method does not work well on problems with highly jumping coefficents. A careful analysis of the twogrid method of the cascadic multigrid method provides a setting where one can understand the difference between the cascadic multigrid method and the classical V-cycle.

On the Treatment of Boundary Conditions in Multiscale Methods for Elliptic P.D.E.'s

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When using wavelet-based multiscale methods for the numerical solution of elliptic partial differential equations, the satisfaction of boundary conditions is usually a delicate question. After collecting some results on the explicit treatment of boundary conditions by Lagrange multipliers, I would like to make some comments on the efficient and fast iterative solution of the resulting linear system. As it turns out, the use of wavelet bases in a preconditioner for the resulting Schur complement yields uniformly bounded condition numbers. In my talk I will focus on the multiscale bases used for this purpose and their construction, both on the domain and the boundary. Alternatively, a ficticious domain approach might be used for the operator part defined on the domain. I would also like to mention some recent results in this direction employing multiscale bases. Finally, I would like to present some recent numerical examples.

A wavelet-Galerkin boundary element method on polyhedral surfaces in $\mathbb{R}^3$

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The implementation of a fast, wavelet-based Galerkin discretization of the double layer potential operator on polyhedral surfaces $\Gamma \subset \mathbb{R}^3$ is described. The algorithm generates an approximate stiffness matrix with $O(N \log^2 N)$ entries in $O(N \log^3 N)$ operations where N is the number of degrees of freedom on the boundary. The condition number of the compressed stiffness matrix is bounded. A C++ realization of the data structure containing the compressed stiffness matrix is described. It can be set up in $O(N \log^2 N)$ operations and requires $O(N \log^2 N)$ memory. Numerical experiments show the asymptotic complexity estimates and convergence rates to be accurate already for moderate N. Problems with $> 10^5$ degrees of freedom were computed in core in less than 1 hour on a workstation. The impact of various parameters in the compression and the quadrature scheme on the performance and the accuracy of the algorithm is studied.

Discretization of Advection on an ARAKAWA-C-grid with Internal Boundaries

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Small-scale meteorological models which simulate the airflow around buildings usually are only able to deal with obstacles that are resolved by the model grid; i.e., a grid box is always either completely covered by a building or completely within the fluid domain. On the other hand, in the last years CFD methods have been developed in aircraft engineering that are able to take into account arbitrarily shaped internal boundaries, although with a rather simple grid structure. We present an adaption of such a method to an Arakawa-C-grid, focussing our attention on the advective processes. In this method, the momentum and scalar fluxes are weighted with the relative coverage of the grid surfaces. The instability problem that arises from cut cells with only a small coverage of free fluid can be avoided by redistributing parts of the flux divergence to the neighboring cells. This advection routine will later on be part of a large numerical model (MITRAS) which will be used to calculate the transport of pollutants in urban areas.

A note on an adaptive nonoverlapping domain decomposition method for elliptic problems

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We consider a nonoverlapping domain decomposition method for elliptic boundary value problems with adaptive interface conditions. The method can be derived from the fictitious overlapping technique (by P.L. Lions). The design of the interface conditions generalizes an idea of Rogier/ Nataf for advection dominated problems.

The method is applicable to the following classes: scalar linear elliptic boundary value problems of second order, linear elasticity problem, linearized Navier–Stokes or Oseen equations (including Stokes).

We prove strong H^1–convergence of the method for the continuous case. Furthermore we present numerical results using a stabilized Galerkin finite element method. The numerical convergence rate is shown to be very reasonable even in case of highly advection or reaction dominated situations.

Discretizations of multi-scale problems in ODEs

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The talk will be about numerical methods for differential equations with different time scales which integrate the equations directly (without analytical preparation).

(a) Stiff integrators have been extensively studied in the past decades for the solution of problems where fast variables are damped out quickly, e.g., parabolic and singularly perturbed problems [1].

(b) Multirate methods are concerned with problems where fast and slow motions are present throughout the integration interval, and a small number (or inexpensive portion) of fast components restricts the step size of standard integrators. The talk presents recent approaches to adaptive inactivation of scales of slow variables [2],[3].

(c) For highly oscillatory problems like Schroedinger or wave equations, where oscillations are generated by the linear part of the equation, we have recently made good experiences with novel integration methods that use the exponential of the Jacobian, with Krylov subspace approxi- mation to the product of the matrix exponential with a vector [4].

[1] HAIRER, E., WANNER, G.: *Solving Ordinary Differential Equations II*, Springer, 1991 (2nd rev. ed. 1996).

[**2**] ENGSTLER, CH., LUBICH, CH.: *Multirate extrapolation for differential equations with different time scales*, SFB 382 Preprint Nr. 29, Univ. Tübingen, 1995.

[**3**] ENGSTLER, CH., LUBICH, CH.: *MUR8: a multirate extension of the eighth-order Dormand-Prince method*, SFB 382 Preprint Nr. 51, Univ. Tübingen, 1996.

[**4**] HOCHBRUCK, M., LUBICH, CH., SELHOFER, H.: *Exponential integrators for large systems of differential equations*, SFB 382 Preprint Nr. 31, Univ. Tübingen, 1995.

Iterative solvers for BEM in 3D with substructuring technique and mixed boundary conditions

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After discretizing, the boundary element equation can be written in matrix form as $Hu = Gt$. Including different types of boundary conditions yields $Ax = b$ where A is generated by corresponding columns of H and G. So the system matrix A is generally fully populated, unsymmetric and not positiv definit. It is well known that convergence of iterative methods highly depends on the structure of A. In [1] it is shown for 2D problems how to apply suitable ordering for columns in A dealing with mixed boundary conditions.
In this work proper ordering schemes and transformations of A are discussed to improve convergence for 3D potential and elasticity problems. Special interest is focused on

- substructuring technique, because standard ordering schemes fail, and on
- mixed boundary conditions, that are described in different coordinate systems for complex geometries. This effect often appears in 3D elasticity where varying types of boundary conditions interact at a single node or element.

The effectivness of the proposed techniques is demonstrated at several iterative methods (CGS, BiCGSTAB, GMRES) with different types of preconditioning (Jacobi, blockdiagonal, Incomplete LU). Numerical results are presented for thermal and elasticity problems with one and more substructures in a range from 400 to 8.000 DOFs.

[1] T.J. Urekew, J.J. Rencis, 'Iterative solution Strategy for BE equations from mixed boundary value problems'. Comput. Methods. Appl. Mech. Eng. **118** (1-2) 13-28 (1994)

A robust multigrid solver for convection-dominated problems

S. Müller

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Standard multigrid solvers are based on an appropriate smoother damping high-oscillatory modes and certain restriction and prolongation operators. Applied to elliptic problems, multigrid algorithms render excellent convergence rates. But those algorithms fail to work, if the convection dominates the flow too much.

I want to present a new strategy how to extend standard multigrid methods to convection-dominated problems. It is based on adapting the restriction and prolongation operators to the differential operator at hand. The construction of the operators can be written in terms of a multiresolution analysis. This tool might be helpful for analytical examinations.

Up to now, test calculations have been performed for

$$\begin{aligned} -\varepsilon \, \Delta \, u + \vec{\beta} \, \nabla \, u &= f \text{ on } \Omega := (0,1)^d \\ u &= 0 \text{ on } \partial\Omega \end{aligned}$$

with $d = 1, 2$. In the one-dimensional case, linear (β =const., $\beta = \beta(x)$) as well as nonlinear problems ($\beta = \beta(u)$) have been examined, whereas for $d = 2$ only the linear coefficient case has been considered. For all computations we obtain convergence rates comparable to those of standard multigrid schemes applied to elliptic problems as long as the grid Peclet number is less than one.

A Numerical Approach to Multiple Scale Problems Based on Asymptotic Analysis

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Multiple scale asymptotic analysis is used as a guideline for developing numerical methods for multiple scale problems. These asymptotic results provide detailed insight into the structure of the solution and are used in the numerical framework. Standard operations of multiple scale asymptotic analysis are introduced as discretized versions of large scale differencing and averaging. This technique is applied to two different problems.

For the self-consistent motion of charged particles in electromagnetic fields described by the Maxwell-Vlasov equations a multiple time scale asymptotic analysis motivates a subcycling procedure to calculate the electron orbits. Two different time scales linked respectively to the electromagnetic wave propagation and the electron gyrofrequency are introduced.

The low Mach number regime of compressible fluid flow is characterized by a large discrepancy between the flow velocity and the speed of sound, leading to physical effects on different length scales and of different orders of magnitude. Acoustic waves with small amplitude but large wave length may interact with local small length scale flow structures while the global background pressure becomes nearly constant in space. A single time scale, multiple space scale asymptotic analysis is used for developing a semi-implicit scheme, which involves multiple pressure variables. Scale adaptive grids are used to capture large and small scale effects.

Optimal error estimates and multiscale applications

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In a model case of an elliptic equation with periodic boundary conditions (or with Dirichlet boundary conditions and cell symmetry) we prove an optimal energy estimate where the right hand side is measured in the L_2-norm. This is transfered to discrete approximations yielding for example convergence of a two-grid scheme for solving the arising linear system. We briefly discuss 'ideal' approximations with adapted function spaces and connections to algebraic multigrid theory.

Geometric modelling of complicated hydrogeologic structures

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A geometric simulator is presented, which is applicable within the framework of hydrogeologic modelling (groundwater flow and contaminant transport).

The data basis for the geometric analysis is obtained from geologic information (maps, vertical profiles, borehole logs). At first this data must be analyzed to construct a consistent geometric model. To this purpose geologic objects (e.g. boreholes, fractures, aquifers) must be identified as lines, surfaces and volumes. Next, topologic consistency must be obtained, that means surfaces and volumes must be compatible (no isolated areas, no penetrating surfaces). A number of tools are implemented in the interactive package DALI to manipulate the data, such as identification functions (boundary generation) and cutting planes, data generalization and mesh generation. The possibilities of the geometric simulator are demonstrated by a number of 2-D and 3-D examples of hydrogeologic systems.

The geometric model provides a first discretization for a numerical analysis. In general, the quality of the discretization must be improved for solving the coupled partial differential equations of flow and transport by use of the finite elment method. The probability of the geometric simulator and the mesh generator is illustrated by a study of a real aquifer system. The data for this investigation was kindly provided by the BGR in Hannover.

Composite Finite Elements for coarse-level discretizations of PDEs with essential boundary conditions

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Composite Finite Elements allow coarse-level discretizations of PDEs where the minimal number of degrees of freedom is independent of the number of micro-structures of the problem (geometric details, oscillating coefficients, etc). We can prove that, for this new finite element space, the approximation property is valid also for the very coarse-level discretizations. In our talk, we will focus on the treatment of essential boundary conditions. For this purpose, we will use prolongation operators which are stable in H^1, are defined locally, and preserve the essential boundary conditions.

Biorthogonal Wavelet Bases on Domains and Manifolds

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Multiscale bases like wavelets offer an versatile and efficient tool for adaptive approximation. For example, they give rise to sparse dicretization for integral equations and promise various application in the numerical solution for partial differential equations. A central problem in this regard is the adaptation of multiscale bases to bounded domains and arbitrary manifolds or surfaces and the incorporation of boundary conditions.

We construct biorthogonal wavelet bases on domains and manifolds Γ, which are given by the union of parametric images of n-dimensional cubes. The order of approximation d and the number of vanishing moments d^* of the primal wavelets can be chosen in a way that $d + d^*$ even. The primal spaces are spline spaces and the regularity of the dual functions is well controlled. Biorthogonality is realized by a modified inner product and the primal as well as the dual functions are globally continuous. Homogeneous Dirichlet boundary conditions can be posed along the boundary of the domain or manifold or along a part of the boundary. A major tool for construction is the *stable completion.* The construction is based biorthogonal wavelets bases on an intervall and tensor product construction for cubes. Afterwards the functions are lifted to the corresponding parametric patches through the parametrization. One can pose either homogeneous boundary conditions or no boundary condition on each face of the cube. Particular attention is paid for gluing the basis functions continuously together.

These multiscale basis functions are appropriate for a conform Galerkin discretization of second order partial differential operators and of (singular) integral operators of nonegative order. They are only suboptimal for operators of order -1. In order to remedy this situation we present a new domain decomposition approach based on multiscale extension operators. With this method one can treat also operators of negative and higher order as well.

Numerical examples are concerning matrix compression for boundary integral equations.

Anisotropic Elements in Linear Elasticity

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We want to solve the equations of linear elasticity on long, but flat domains. Due to the degeneracy of the constant in Korn's inequality a straight forward finite element method with anisotropic elements gives bad approximation (locking).

The standard techniques to handle these beams and plates are models from structural mechanics. The 2D (3D) displacement functions are first approximated by functions of a lower dimensional function space, e.q. functions fulfilling the Kirchhoff hypotheses. Then the reduced model is discretized by finite elements.

In this talk, a non-conforming discretization method of the original linear elasticity problem is introduced. The discrete solution is equivalent to the discrete solution of the structural mechanics problem. Therefore no locking occurs. The implementation of the non-conforming method seems to be much simpler, especially when different models are coupled.

The convergence rate of multigrid solvers with point-Gauss-Seidel smoothers depends on the geometric parameters. A block-Gauss-Seidel smoother leads to a robust multigrid method.

Numerical examples demonstrate the accuracy of the discretization and the robustness of the solver.

BEM Solution of Laplace's Equation in Multi-LayeredMedia with High Permittivity Ratios

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A boundary integral formulation for Laplace's equation $\nabla \cdot a\nabla u = 0$ with piecewise constant coefficients a is discussed. Although the arising integral operator and its inverse can be bounded independently of the coefficients, numerical problems occur when the dynamic range of the coefficients is large. This is due to different scaling of the solution of the integral equation which results in large discretization errors of the flux $a\nabla u$. We will discuss a novel two step technique to avoid different scaling. The analysis of this approach shows that the discretization error and the condition of the discretized linear system can be bounded independently of the coefficients. Numerical experiments concerning the three-dimensional capacitance extraction of structures with multiple dielectrics will demonstrate the accuracy and efficiency of the method.

Three-dimensional multigrid based pressure solver for the computation of the flow around the HSVA Tanker

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We consider three-dimensional numerical methods for the incompressible Reynolds averaged Navier-Stokes equations discretised by finite difference techniques on non-staggered grids in boundary fitted coordinates. A segregated approach is used to solve the pressure-velocity coupling problem, whose resolution of pressure Poisson equation -a crucial task especially for three-dimensional complex geometries- is handled by a multigrid based solver.

We present the numerical treatment and implementation aspects of this solver and show its performance on the computation of the turbulent viscous flow around the HSVA tanker at high Reynolds number (five millions) [1]. An elliptic pressure equation with highly varying coefficients mainly involved by the topology of the mesh ($96\times48\times32$, non-orthogonal and stretched near the hull) is to be solved at each non-linear iteration. The linear cell-centered multigrid employs the Galerkin Coarse Grid Approximation to build the matrices on coarser grids [2]. The cell-centered layout avoids the use of matrix-dependent operators. The uniform operators (linear interpolation for the prolongation, arithmetical average for the restriction) ensures the optimal sparsity of the stencil (7 points) on each grid and simultaneously allows to keep the M-matrix property for the pressure operator on each grid. As smoother, we employ the Strongly Implicit Procedure of Stone [3].

Using this multigrid method as a preconditioner for the BICGSTAB algorithm [4] results in a CPU-time speed up ratio greater than two with regard to other conventional acceleration techniques (MILU preconditioned Krylov subspace methods) on this problem. According to the aforementioned choices of transfer operators and smoothing strategy, this multigrid based solver is the starting point for a parallelizable tool well-suited to a multiblock approach.

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Filter Bank Methods for Hyperbolic PDEs

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We use biorthogonal filter banks to solve hyperbolic PDEs adaptively with a sparse multilevel representation of the signal. The methods described are of finite difference type, and the filter banks are used to give a sparse representation of signals, and to transform between grids on different scales. We derive bounds for the error and number of coefficients in the sparse representation. These bounds also apply for filter banks that are not associated with any wavelets. The strength of the method is shown in various test problems.

Multi-grid for plate bending in scattered data interpolation

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We consider the scattered data interpolation problem. Given a set of function values in a two dimensional domain, one wants to construct a smooth global interpolation function through the data. The interpolant is determined by a curvature minimization criterion, fulfilling the Kirchhoff plate bending or biharmonic equation. This construction is analogue to the curvature minimization property of one-dimensional splines. The approach is motivated by an implementation of the plate bending interpolation method in the commercial package Siscat (Sintef Scattered Data Library).

We construct a multi-grid method for the biharmonic equation. The problem is the correct treatment of the interpolation conditions represented as prescribed function values in single points. Usually the interpolation points are not present on a coarse grid, since the position of the interpolation points is completely random. We construct the coarse grid matrices by means of Galerkin products which implements the necessary modifications in case of prescribed points on the finest grid. The restriction and prolongation is modified accordingly.

The Proceedings are published in the series

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